

Unit 2: Exponential and Logarithmic Functions

Unit 2 covers exponential and logarithmic functions: how to tell exponential growth apart from linear growth, rewriting equivalent exponential forms, compound and continuous growth, the definition and properties of logarithms, solving exponential and logarithmic equations, semi-log plots, and logistic models.

AP Precalculus

Exponential and Logarithmic Functions

About 13 minutes to read

How to use this guide

Read it in order the first time because the topics build on each other. Exponential growth motivates the need for logarithms, the properties of logarithms let you rewrite expressions, and rewriting is what makes exponential and logarithmic equations solvable. Semi-log plots and logistic models then apply the same ideas to real data.

After the first read, use the trap boxes and the confusions table to review the distinctions that exam questions test most often. Finish with the practice questions, then complete the recall check on the last page out loud and note any items you cannot explain yet.

What this unit is worth. Exponential and Logarithmic Functions is about 25 to 40 percent of the AP Precalculus exam, which makes it the largest unit on the test. It is also the unit most likely to show up inside other topics, since exponential models appear in data analysis and logarithmic scales appear across the sciences.

2.1 Exponential vs Linear Growth

A linear function grows by **constant difference**. Each step forward adds the same amount: $f(x) = mx + c$ has a slope of m , so every unit increase in x raises the output by m . An exponential function grows by **constant percent change**. Each step forward multiplies the output by the same factor: $f(x) = a \cdot b^x$ has a growth factor b , so every unit increase in x multiplies the output by b . Growth means $b > 1$; decay means $0 < b < 1$.

The table below compares $E(x) = 20(1.1)^x$, which grows 10% per step, with $L(x) = 4x + 20$, which adds 4 per step. For the exponential column, each entry is 1.1 times the one before: $20 \times 1.1 = 22$, $22 \times 1.1 = 24.2$, $24.2 \times 1.1 = 26.62$, $26.62 \times 1.1 = 29.282$. For the linear column, each entry is 4 more than the one before. Constant ratio marks exponential growth; constant difference marks linear growth.

x	$E(x) = 20(1.1)^x$	$L(x) = 4x + 20$
0	20	20
1	22	24
2	24.2	28
3	26.62	32
4	29.282	36

Trap. A constant percent change is not a constant amount. Growing by 10% per step means the added amount itself grows: $E(x)$ adds 2, then 2.2, then 2.42, then 2.662 as x goes from 0 to 4. Questions that ask "how much is added each step" are testing whether you notice the amounts changing in an exponential model.

2.2 Equivalent Forms of Exponential Functions

The same growth can be written over different time units, and the rates convert through the **equivalence** $(1 + r_{\text{annual}}) = (1 + r_{\text{monthly}})^{12}$. If a quantity grows 7% per year, the monthly growth factor r_m satisfies $(1 + r_m)^{12} = 1.07$. Taking the 12th root: $r_m = 1.07^{1/12} - 1$. Since $1.07^{1/12} \approx 1.005654$, the monthly rate is about 0.005654, or 0.5654%. Check the answer by compounding it back: $1.005654^{12} = 1.07$, which matches the annual factor exactly.

Notice that 0.5654% is not $7/12 \approx 0.5833\%$. Dividing the rate by 12 ignores compounding, which is the whole point of the unit. The same idea lets you rewrite any exponential form: $5^{2x} = 25^x$, and a function given as $f(t) = 100(2)^{t/12}$ doubles every 12 time units because the exponent reaches 1 when $t = 12$.

Trap. Do not convert rates by simple division. A 7% annual rate is not a 0.5833% monthly rate; it is 0.5654% monthly, because the monthly rate must compound twelve times back up to the annual factor of 1.07. Any option that divides by 12 is ignoring compounding.

2.3 Compound Interest

When interest compounds n times per year at annual rate r , the balance after t years is $A = P(1 + r/n)^{nt}$. The exponent nt counts the total number of compounding periods, and r/n is the rate per period. The formula requires r as a decimal, so 5% enters as 0.05.

Worked example: \$1,000 invested at 5% compounded quarterly for 10 years. Here $P = 1000$, $r = 0.05$, $n = 4$, $t = 10$, so the per-period rate is $0.05/4 = 0.0125$ and the number of periods is $4 \times 10 = 40$. The balance is $A = 1000(1.0125)^{40}$. Since $1.0125^{40} \approx 1.643619$, the account holds about \$1,643.62. To sanity-check the exponent, note that quarterly compounding over 10 years must involve exactly 40 growth steps.

Trap. The exponent is nt , the total number of periods, not t . Quarterly compounding for 10 years uses $(1.0125)^{40}$, not $(1.0125)^{10}$. The other classic error is entering the rate as 5 instead of 0.05, which blows the balance up to an absurd value.

2.4 The Number e and Continuous Growth

The number $e \approx 2.71828$ is what the expression $(1 + 1/n)^n$ approaches as n grows without bound. Intuitively, e is the result of compounding 100% growth infinitely often. When compounding is continuous, the compound interest formula $A = P(1 + r/n)^{nt}$ converges to $A = Pe^{rt}$, the continuous growth model.

Worked example: \$1,000 at 5% compounded continuously for 10 years. Here $A = 1000e^{0.05 \times 10} = 1000e^{0.5}$. Since $e^{0.5} \approx 1.648721$, the balance is about \$1,648.72. Compare with the quarterly result of \$1,643.62 from the previous section: continuous compounding earns \$5.10 more, exactly as you would expect, since compounding more often can only raise the balance for a positive rate.

Trap. Pe^{rt} is the continuous model; $P(1 + r)^t$ is annual compounding. A question that says "compounded continuously" wants the e form, and the exponent is the plain product rt , with no n involved.

2.5 Logarithms: Definition and Evaluation

The statement **$\log_b(x) = y$ means $b^y = x$** , with $b > 0$, $b \neq 1$, and $x > 0$. The logarithm answers the question "to what power must I raise b to get x ?" Two special cases have their own names: $\log_{10}$ is written as $\log$, and $\log_e$ is written as $\ln$, the natural logarithm.

To evaluate a logarithm by hand, convert it to the exponential form and solve for the power. Check each answer by confirming $b^y = x$.

- $\log_2(32) = 5$, because $2^5 = 32$. Confirm: $2 \times 2 \times 2 \times 2 \times 2 = 32$.
- $\log_5(1/25) = -2$, because $5^{-2} = 1/5^2 = 1/25$. Confirm: $1/25 = 0.04$, and $5^{-2} = 0.04$.
- $\log_3(81) = 4$, because $3^4 = 81$. Confirm: $3 \times 3 \times 3 \times 3 = 81$.
- $\log_{10}(0.001) = -3$, because $10^{-3} = 1/1000 = 0.001$. Confirm: $1/10^3 = 1/1000$.

Three values are worth memorizing because they fall out of the definition directly: $\log_b(1) = 0$ for any base, since $b^0 = 1$; $\log_b(b) = 1$, since $b^1 = b$; and $\ln(e) = 1$, since $e^1 = e$.

Trap. A logarithm of zero or a negative number is undefined. The domain condition $x > 0$ in the definition is what creates extraneous solutions in Section 2.7, so take it seriously now.

2.6 Properties of Logarithms

The log properties turn the exponent rules into addition, subtraction, and multiplication. For $x > 0$ and $y > 0$: the **product rule** $\log_b(xy) = \log_b x + \log_b y$, the **quotient rule** $\log_b(x/y) = \log_b x - \log_b y$, and the **power rule** $\log_b(x^p) = p \cdot \log_b x$. The **change of base** formula $\log_b x = \ln x / \ln b$ lets a calculator evaluate any base.

Worked example, expanding step by step: $\log_5(25x^2/y)$. First apply the quotient rule to split the division: $\log_5(25x^2) - \log_5 y$. Then the product rule: $\log_5 25 + \log_5(x^2) - \log_5 y$. Then the power rule: $\log_5 25 + 2 \log_5 x - \log_5 y$. Since $\log_5 25 = 2$, the expansion is $2 + 2 \log_5 x - \log_5 y$.

Verify it numerically with $x = 10$ and $y = 5$. The original expression is $\log_5(25 \times 100 / 5) = \log_5(500)$. By change of base, $\log_5(500) = \ln(500) / \ln(5) = 6.214608 / 1.609438 \approx 3.861353$. The expanded form gives $2 + 2 \log_5(10) - \log_5(5)$. Since $\log_5(10) = \ln(10)/\ln(5) \approx 1.430677$ and $\log_5(5) = 1$, this equals $2 + 2(1.430677) - 1 = 3.861353$. Both sides agree.

Trap. Logarithms do not distribute over sums: $\log(a + b)$ is not $\log a + \log b$. The product rule applies to multiplication inside the log, $\log(ab) = \log a + \log b$, and nothing similar exists for addition. Watch for this in multiple-choice distractors.

2.7 Solving Exponential and Logarithmic Equations

To solve an exponential equation, isolate the power and take a logarithm of both sides. For $2^{3x-1} = 50$, taking $\log_2$ of both sides gives $3x - 1 = \log_2(50)$. Solving for x : $x = (1 + \log_2 50) / 3$. Evaluate the log with change of base: $\log_2(50) = \ln(50) / \ln(2) = 3.912023 / 0.693147 \approx 5.643856$. Then $x = (1 + 5.643856) / 3 = 6.643856 / 3 \approx 2.214619$.

Verify by substituting back into the original equation. The exponent becomes $3(2.214619) - 1 = 6.643856 - 1 = 5.643856$. Then $2^{5.643856} = e^{5.643856 \times \ln 2} = e^{5.643856 \times 0.693147} = e^{3.912023} = 50$, since $e^{3.912023}$ is $e^{\ln 50}$. The solution checks out.

Logarithmic equations need a **domain check** on every candidate solution, because the log rules can introduce roots that make an argument zero or negative. Solve $\log x + \log(x - 3) = 1$ (base 10). Combine with the product rule: $\log(x(x - 3)) = 1$. Convert to exponential form: $x(x - 3) = 10^1 = 10$, so $x^2 - 3x - 10 = 0$, which factors as $(x - 5)(x + 2) = 0$. The candidates are $x = 5$ and $x = -2$.

Check $x = 5$: $\log 5 + \log 2 = \log(5 \times 2) = \log 10 = 1$, so $x = 5$ works. Check $x = -2$: the original arguments are $\log(-2)$ and $\log(-5)$, both undefined, so $x = -2$ is an **extraneous solution** and must be rejected. The equation has exactly one solution, $x = 5$.

Trap. The factored candidates are not the answer until each one is substituted back. Combining logs with the product rule assumes both arguments are positive, an assumption the algebra then forgets. Always test each candidate in the original equation.

2.8 Semi-Log Plots

If data follows an exponential model $y = ab^x$, taking the log of both sides straightens it into a line. Using log base 10: $\log y = \log(a \cdot b^x) = \log a + x \log b$. This has the form of a line in x with **slope $\log b$** and y -intercept $\log a$. That is the point of a semi-log plot, which graphs $\log y$ against x : exponential data appears as a straight line, and the slope of that line reveals the growth rate.

Read the slope carefully. For $y = 4(3)^x$, the semi-log equation is $\log y = \log 4 + x \log 3$, so the slope on a base-10 plot is $\log 3 \approx 0.4771$, not 3. A positive slope means $b > 1$ (growth); a negative slope means $b < 1$ (decay). The intercept gives $\log a$, so $a = 10$ raised to the intercept value.

Trap. The semi-log slope is $\log b$, not b . A slope of 0.4771 on a base-10 semi-log plot means the growth factor is 3, because $\log 3 \approx 0.4771$. Answering 0.4771 when the question asks for the growth factor confuses the log of the rate with the rate itself.

2.9 Logistic Models

A **logistic model** describes growth that is limited by a ceiling, the **carrying capacity** L : $P(t) = L / (1 + Ce^{-kt})$. Early on the model looks exponential, then growth slows as the population approaches L , and the curve flattens out at L . The initial value is $P(0) = L / (1 + C)$, and the growth is fastest at the **inflection point**, which occurs when $P = L/2$, half the carrying capacity.

Example: $P(t) = 1000 / (1 + 9e^{-0.5t})$. The carrying capacity is 1000, the initial population is $P(0) = 1000 / (1 + 9) = 100$, and the inflection point is at $P = 500$. To find when that happens, solve $500 = 1000 / (1 + 9e^{-0.5t})$: dividing gives $1 + 9e^{-0.5t} = 2$, so $9e^{-0.5t} = 1$, $e^{-0.5t} = 1/9$, and $-0.5t = -\ln 9$, giving $t = 2 \ln 9 \approx 4.394$. Check: at $t \approx 4.394$, $e^{-0.5t} = e^{-\ln 9} = 1/9$, so $P = 1000 / (1 + 1) = 500$, confirming the inflection.

Trap. Logistic growth is bounded by its carrying capacity. Unlike pure exponential growth, a logistic curve never exceeds L , no matter how much time passes. Questions that ask for the long-term value want L , not infinity.

Confusions That Cost Points

PAIR	HOW TO KEEP THEM STRAIGHT
$\log(a + b)$ vs $\log a + \log b$	There is no rule for a sum inside a log. The product rule says $\log(ab) = \log a + \log b$, with multiplication inside. If you see addition inside a log, leave it alone.
Exponential b^x vs power x^b	In an exponential function the variable is the exponent; in a power function the variable is the base. $f(x) = 2^x$ grows by constant percent change, while $f(x) = x^2$ does not.
a^{b+c} vs $a^b + a^c$	Adding exponents means multiplying powers: $a^{b+c} = a^b \cdot a^c$, not $a^b + a^c$. This is the exponential version of the log-sum trap.
Extraneous solutions vs valid ones	A log equation can produce candidates that make an argument zero or negative. Substitute every candidate into the original equation and reject any that break the domain.
Percent change vs percentage-point change	A 7% annual rate converts to a 0.5654% monthly rate through $(1.005654)^{12} = 1.07$, not to $7/12 \approx 0.5833\%$ by division. Compounding is what connects the two rates.

Practice Questions

Original questions written for this guide in the style of the AP exam. Answers and explanations are on the next page, so complete the questions before checking them.

1. Which of the following functions grows exponentially?

1. $f(x) = 5x + 1$
2. $f(x) = 5(1.3)^x$
3. $f(x) = 5x^2$
4. $f(x) = 5/x$

2. A quantity grows at an annual rate of 6%. What is the equivalent monthly growth rate?

1. 0.50%
2. 0.06%
3. 0.005%
4. 0.49%

3. \$2,000 is invested at 4% compounded monthly for 5 years. What is the balance at the end?

1. \$2,400.00
2. \$2,433.31
3. \$2,441.99
4. \$2,426.44

4. A population of 500 grows continuously at 8% per year. Which expression gives its size after 3 years?

1. $500e^{0.24}$
2. $500(1.08)^3$
3. $500 + 500(0.08)(3)$
4. $500/e^{0.24}$

5. $\log_4(1/16) =$

1. 4
2. -4
3. -2
4. 2

6. Which of the following is equivalent to $\log_3(x^2y / z)$ for $x, y, z > 0$?

1. $(2 \log_3 x)(\log_3 y) / \log_3 z$
2. $\log_3 x^2 + \log_3 y + \log_3 z$
3. $(2 \log_3 x + \log_3 y) / \log_3 z$
4. $2 \log_3 x + \log_3 y - \log_3 z$

7. Solve $3^x = 20$ for x .

1. 2.996
2. 2.727
3. 6.667
4. 0.367

8. Solve $\log_2 x + \log_2(x - 2) = 3$.

1. $x = 4$
2. $x = -2$
3. $x = 4$ or $x = -2$
4. $x = 2$

Answer Key

- 1. B.** Only B has the variable in the exponent, $f(x) = 5(1.3)^x$, which is the exponential form $a \cdot b^x$. A is linear, growing by a constant difference. C is a power function, with the variable as the base, not the exponent. D is a reciprocal function, which decays toward zero rather than multiplying by a constant factor.
- 2. D.** The monthly rate must satisfy $(1 + r_m)^{12} = 1.06$, so $r_m = 1.06^{1/12} - 1 \approx 0.004868$, or 0.49%. A divides 6% by 12 and ignores compounding. B is 6/100 written as a percent by mistake. C converts to a decimal twice, dividing an already-decimal rate by 12 and calling the result a percent.
- 3. C.** Monthly compounding means $n = 12$: $A = 2000(1 + 0.04/12)^{60} = 2000(1.003333)^{60} \approx \$2,441.99$. A uses simple interest, $2000(1 + 0.04 \times 5) = \$2,400$. B compounds annually, $2000(1.04)^5 \approx \$2,433.31$. D has no consistent compounding scheme behind it.
- 4. A.** Continuous growth uses $A = Pe^{rt}$ with $rt = 0.08 \times 3 = 0.24$, giving $500e^{0.24} \approx 635.62$. B compounds once per year instead of continuously. C adds a fixed amount each year, which is linear, not exponential. D puts the growth in the denominator, which models decay rather than growth.
- 5. C.** Set $4^y = 1/16$. Since $4^2 = 16$, $4^{-2} = 1/16$, so $y = -2$. A and B solve $4^y = 256$ and $4^y = 1/256$ by misreading the argument. D flips the sign, answering $4^2 = 16$ for a question about $1/16$.
- 6. D.** Apply the quotient rule, then the product and power rules: $\log_3(x^2y/z) = \log_3(x^2y) - \log_3 z = 2 \log_3 x + \log_3 y - \log_3 z$. A multiplies the logs instead of adding them. B adds $\log_3 z$ instead of subtracting it. C divides by $\log_3 z$, as if the quotient rule distributed the division.
- 7. B.** Taking logs gives $x = \log_3(20) = \ln 20 / \ln 3 = 2.995732 / 1.098612 \approx 2.727$. A computes $\ln 20 \approx 2.996$ but forgets to divide by $\ln 3$. C divides the argument by the base, $20/3$, which is not a log operation. D takes the reciprocal of the correct answer.
- 8. A.** Combine: $\log_2(x(x - 2)) = 3$, so $x(x - 2) = 8$, $x^2 - 2x - 8 = 0$, and $(x - 4)(x + 2) = 0$. The candidates are 4 and -2, but $x = -2$ makes both log arguments negative, so it is extraneous. Only $x = 4$ survives: $\log_2 4 + \log_2 2 = 2 + 1 = 3$. B keeps the extraneous root. C fails to check the domain. D makes the argument $x - 2$ equal to zero, which is undefined.

When you check your answers, note which distinction each miss came from. Make a flashcard for that distinction and drill it spaced out over the next few days instead of rereading the whole section. If you missed one of these questions, the same distinction is worth practicing again in Rycal, where the Exponential and Logarithmic Functions deck has flashcards for it and more practice questions use the same kinds of traps.

One-Page Recall Check

Say each answer out loud before you look back, and mark the ones you cannot finish. Anything you cannot say out loud yet belongs in your flashcard deck. In Rycal, add those items to the Exponential and Logarithmic Functions deck and let spaced review bring them back over the next few days.

- Explain the difference between constant percent change and constant difference, with an example of each.
- Convert a 7% annual growth rate to a monthly rate, showing the equation you solve.
- State the compound interest formula and explain what n and the exponent nt count.
- Compute \$1,000 at 5% compounded quarterly for 10 years, showing each step.
- Explain where the number e comes from and state the continuous growth formula.
- State the definition of $\log_b(x) = y$ in terms of exponents, including the domain restrictions.
- Evaluate $\log_2(32)$, $\log_5(1/25)$, and $\log_3(81)$ by hand, checking each with $b^y = x$.
- State the product, quotient, and power rules and the change-of-base formula.
- Expand $\log_5(25x^2/y)$ step by step.
- Solve $2^{3x-1} = 50$ and verify the solution by substitution.
- Solve $\log x + \log(x - 3) = 1$ and explain why one candidate is rejected.
- Explain why a semi-log plot of exponential data is a straight line and what its slope equals.
- Describe a logistic model: carrying capacity, initial value, and where growth is fastest.

Where to go next. Turn every missed item above into flashcards and drill them spaced out over several days rather than in one sitting. In Rycal, open the Exponential and Logarithmic Functions deck under AP Precalculus. The deck covers the terms in this guide, and its practice questions target the same traps named here. If you have a test date, add it in the Test Planner. You can also start your next review with a Brain Dump, then check what you missed against this guide.

Key terms for this unit

Constant percent change, Constant difference, Growth factor, Equivalent exponential forms, Compound interest, Continuous compounding, The number e , Logarithm, Common log, Natural logarithm, Product rule, Quotient rule, Power rule, Change of base, Extraneous solution, Semi-log plot, Logistic model, Carrying capacity, Inflection point.

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